International Journal of Heat and Mass Transfer 52 (2009) 796–804

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Assessment of structure effects on the thermal conductivity of two-phase porous geomaterials

Jean Côté *, Jean-Marie Konrad [1]

Département de Génie Civil, Faculté des Sciences et de Génie, 1065, Avenue de la Médecine, Pavillon Adrien-Pouliot, Local 2912A, Université Laval, Québec, Canada G1V 0A6

ARTICLE INFO

Article history:
Received 13 November 2007
Received in revised form 11 July 2008
Available online 20 September 2008

Keywords:
Thermal conductivity
Solid
Fluid
Porosity
Particles shape
Cement

ABSTRACT

The effect of structure on the thermal conductivity of geomaterials is studied for solid–fluid combinations representing a wide variety of two-phase porous geomaterials. Nearly 200 thermal conductivity data sets from the literature were analyzed for geomaterials made of natural soil particles, crushed rock particles and sedimentary rock. Two analog models are studied to quantify the effect of structure. It appears that the effect of structure increases with decreasing fluid/solid thermal conductivity ratio and structure effects are negligible from a ratio of approximately 1/15 and higher. A new simplified model is proposed to compute the effective thermal conductivity as a function of the fluid/solid thermal conductivity ratio and the structure of geomaterials. The model applies well to independent data of homogeneous and heterogeneous materials including industrial cement concrete.

1. Introduction

Thermal analyses of civil engineering infrastructures require the knowledge of the effective thermal conductivity of geomaterials that include natural soil, natural rock, crushed rock, cement concrete and bitumen concrete among others. The thermal conductivity of soils is governed by many parameters such as porosity, water content, mineral content and grain size distribution. Several studies have also recognized the structure effects owed to cementation [1,2] and to particle shape [3–6] especially in dry conditions (saturated with air). However, studies on the quantitative assessment of structure effects generally only cover idealized (or hypothetical or theoretical) soils.

The objective of this paper is thus to develop new semi-empirical modeling approaches to assess the effect of structure on the effective thermal conductivity of two-phase porous geomaterials. The paper first describes the structure of porous geomaterials and its effect on the effective thermal conductivity. Following is a background on the thermal conductivity bounds for two-phase materials and on different modeling approaches. The paper uses two analog models to assess the effect of structure on the effective thermal conductivity of two-phase porous geomaterials: Fricke's model [7] based on theoretical geometrical assumptions and the Côté and Konrad model [6] based on the relative thermal conductivity concept and the empirical assessment of structure effects. The analysis results of nearly 200 thermal conductivity data sets are discussed in terms of structure effects and a new simplified model is proposed and validated.

2. Structure of two-phase porous geomaterials

Geomaterials used in civil engineering infrastructures are mostly made of mineral particles of various shapes and size distributions. Inter-particle pores are generally filled with water and/or air. When the pores are saturated with either water or air, the materials are considered as two-phase porous geomaterials. The particles of geomaterials can also be bound together as a result of natural or industrial processes. The porous geomaterials are thus highly heterogeneous and may even be anisotropic depending on shape and orientation of particles. According to Johansen [4], particle and pore size distributions of most geomaterials allow heat transfer by conduction only. Convection and radiation effects are thus neglected in this study.

Heat conduction through two-phase porous media depends on the thermal conductivity of each phase and on the structure of the solid matrix. In terms of thermal behavior, the structure of the solid matrix characterizes the contact resistance and the continuity of the solid phase [8]. In geotechnical engineering the structure of geomaterials is generally associated with the combined effect of fabric, soil composition and interparticle forces. The term fabric refers to the arrangements of particles, particle groups and pore space [9]. It is recognized that the fabric and the structure

* Corresponding author. Tel.: +1 418 656 5936; fax: +1 418 656 2928.
E-mail addresses: Jean.Cote@gci.ulaval.ca (J. Côté), Jean.Marie-Konrad@gci.ulaval.ca (J.-M. Konrad).
[1] Tel.: +1 418 656 3878.

0017-9310/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2008.07.037

Nomenclature

a, b, c	axis of spheroids
d	particle diameter
F	weighting factor
g	shape factor
HSL	Hashin/Shtrikman lower bound
HSU	Hashin/Shtrikman upper bound
K	effective thermal conductivity
k_{dry}	thermal conductivity of dry soil (air saturated)
k_f	thermal conductivity of fluids
k_r	relative thermal conductivity
k_s	thermal conductivity of solids
k_{sat}	thermal conductivity of water saturated soil
n	porosity = v_v/v_t
S_r	degree of saturation = v_w/v_v
v_v	volume fraction of voids
v_w	volume fraction of water
v_t	total volume
WL	Weiner lower bound
WU	Weiner upper bound
Z	c/a
Greek symbols	
β	empirical parameter
κ	weighting factor (moist soils)
κ_{2P}	weighting factor (two-phase porous geomaterials)

of geomaterials are influenced by the size and shape of particles and the aggregation of particles with or without cement/bounding agents, which can be of natural origin (calcite, silicates, iron oxides, phosphate, etc.) or of industrial origin (bitumen, lime, Portland cement).

The particle shapes and the presence of bounding agents will thus influence the degree of contact resistance and the continuity of the solid phase of the porous geomaterials, which in turn, influence its effective thermal conductivity. Rounded and sub-rounded particles are found in natural soil deposits while angular and sub-angular particles are generally obtained from the rock crushing operations. Sedimentary rocks are naturally cemented geomaterials, while cement or bitumen concretes are industrially cemented geomaterials. Therefore, in terms of thermal characteristics, the structure of porous geomaterials can be divided in three categories: (a) unbound rounded/sub-rounded particles, (b) unbound angular/sub-angular particle and (c) bound/cemented particles as shown in Fig. 1. The solid-to-solid contact area of the unbound angular particles is greater than that of the unbound rounded particles, while the solid-to-solid contact area is further increased when bounding agents are present. The bound/cemented particles category includes round/sub-rounded and angular/sub-angular particles as it is expected that the effect of angularity of particles decreases substantially in the presence of cement.

Studies of the effect of structure on heat flow and effective thermal conductivity (k) of two-phase porous geomaterials have shown that k is generally higher in cemented materials than in loose particle packs [1,2,4]. Furthermore, it was observed that the influence of structure on the effective thermal conductivity of two-phase porous media increases with decreasing solid/fluid thermal conductivity ratios [2]. Hamilton and Crosser [10] also showed theoretically that the k values of particle packs decreases with increasing sphericity of particles. This was observed by Johansen [4] and by Côté and Konrad [6] in air-saturated porous geomaterials where the k values of natural particle packs (rounded/sub-rounded) were systematically lower than those of crushed particle packs (angular/sub-angular). Recently, the fast growing performances of desktop computers enabled the study of structure effects on the effective thermal conductivity of various porous media also showing the importance of particle contact density [11,12].

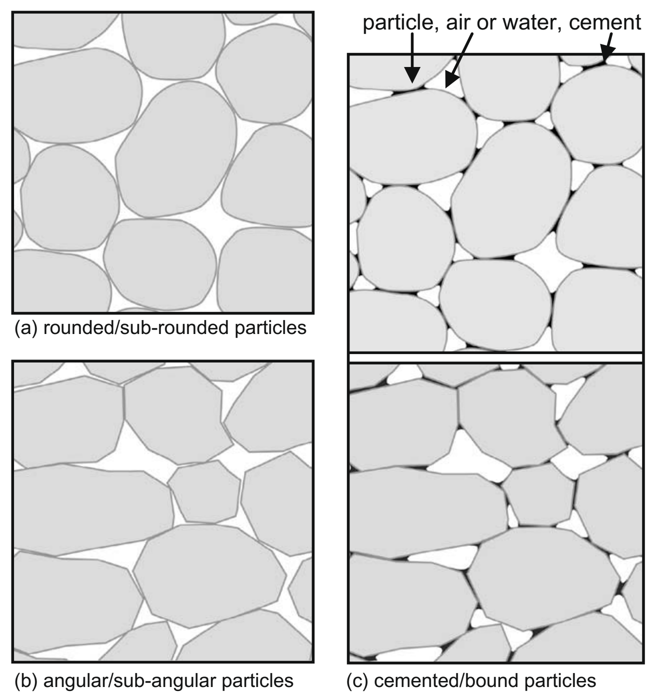

Fig. 1. Schematic structures of porous geomaterials.

3. Thermal conductivity bounds and modeling approaches

3.1. Thermal conductivity bounds

Independently of structure effects, the series and parallel flow models are universally accepted as yielding the widest bounds, also known as the Weiner bounds, in between which the effective thermal conductivity of two-phase materials should stand. The parallel flow model (Eq. (1)) is the upper bound (WU) while the series flow model (Eq. (2)) provides the lower bound (WL):

$$k = (1 - n)k_s + nk_f \tag{1}$$

$$k = \frac{k_s k_f}{(1 - n)k_f + nk_s} \tag{2}$$

Hashin and Shtrikman [13] proposed narrower bounds for homogeneous isotropic two-phase materials. Their upper bound (HSU) applies to a continuous solid phase including uniformly dispersed fluid filled cavities (Eq. (3)) and their lower bound (HSL) applies to a continuous fluid phase including uniformly dispersed solid spheres (Eq. (4)). The HSU and HSL bounds are mathematically equivalent to the well known Maxwell model [14] and thus represent the internal and external porosity cases [15]:

$$k = k_s + \frac{3nk_s(k_f - k_s)}{3k_s + (1-n)(k_f - k_s)} \quad (3)$$

$$k = k_f + \frac{3(1-n)(k_s - k_f)}{3k_f + (1-n)(k_s - k_f)} \quad (4)$$

The various two-phase porous geomaterials used in civil engineering are heterogeneous with different particle contact density. Neither phase is necessarily continuous or dispersed; either phase may thus form continuous heat conduction pathways [15]. Numerous studies have shown that geomaterials made of rounded/sub-rounded particles have thermal conductivities neighboring the HSL bound resulting from the low particle contact density and limited amount of continuous solid pathways. It is thus expected the k values for geomaterials made of angular/sub-angular particles and cement/bound particles will move away from the HSL bound as a result of the increasing contact area and continuity of the solid phase.

It is noted that narrower bounds were also proposed on the basis of volume fractions and thermal conductivity of each phase [16] and particle sphericity [17], while Carson et al. [15] divided the Hashin/Shtrikman bounds in two regions according to the concept of internal/external porosity of materials.

3.2. Theoretical versus empirical modeling approaches

Various geometrical hypotheses were used to express combined parallel and series heat flow theoretical model for unit cells such as that of Mickley [18] and Crane and Vachon [19] among many others while Hadley [20] and Samantray et al. [21] used empirically weighted averages of the Hashin and Shtrikman bounds to model the effective thermal conductivity of numerous two-phase porous media. Wang et al. [22] developed a unifying equation for composites structures of different combination of the Weiner and Hashin/Shtrikman bounds and the effective medium theory (EMT) providing unlimited possibilities for theoretical models. Since the structures of natural of industrial geomaterials are highly variable, the choice of adequate composite structures (Wang's model) or their equivalent weighting factors (Hadley's and Samantray's model) remains a speculative task, therefore making the extension of any theoretical model to heterogeneous media a fairly complex process. Semi empirical approaches may still remain more convenient and efficient [20].

Progelhof et al. [23] reviewed more than 20 empirical and theoretical models and also concluded that the semi-empirical approach may be more appropriate to characterize the effective thermal conductivity of structured materials such as low density foams. In an attempt to characterize the effect of structure on the effective thermal conductivity of two-phase porous geomaterials, this paper will therefore use a semi-empirical approach based on the theoretical model developed by Fricke [7] and the relative thermal conductivity model by Côté and Konrad [6].

4. Fricke's approach

The theoretical model developed by Fricke [7], as extended by DeVries [24], has been fairly successful in the estimation of the effective thermal conductivity of geomaterials such as soils. Fricke's model originates from the Poisson's theory of induced magnetism for suspension of homogeneous solid spheres (no solid-to solid contacts). Fricke [7] extended it to consider a homogeneous suspension of arbitrarily oriented oblate/prolate spheroids. The a ad b axes of the spheroids are of equal length and the poles are on c-axis (see spheroids shown in Fig. 2). Following these premises, the effective thermal conductivity is expressed by:

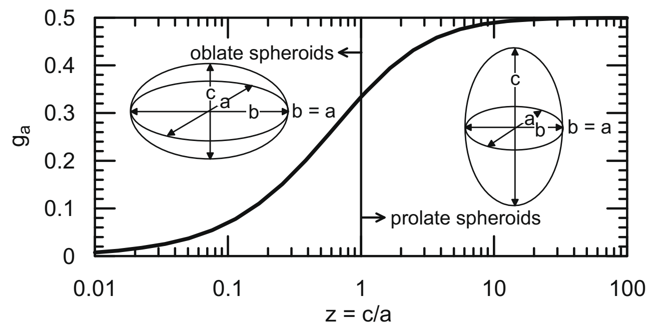

Fig. 2. Shape factor g_a for oblate/prolate spheroids.

$$k = \frac{nk_f + (1-n)Fk_s}{n + (1-n)F} \quad (5)$$

with

$$F = \frac{1}{3}\left[\frac{2}{1 + (k_s/k_f - 1)g_a} + \frac{1}{1 + (k_s/k_f - 1)(1 - 2g_a)}\right] \quad (6)$$

For oblate spheroids (flattened at the poles, $c < a$) g_a is given by Eqs. (7) and (8) is for prolate spheroids (elongated at the poles $c > a$), with $z = c/a$:

$$g_a = \frac{z}{2}\frac{\cos^{-1}(z) - z\sqrt{1-z^2}}{\sqrt{(1-z^2)^3}} \quad (7)$$

$$g_a = \frac{1}{2(1-z^{-2})} - \frac{z^{-2}}{4\sqrt{(1-z^{-2})^3}} \ln\left[\frac{1+\sqrt{1-z^{-2}}}{1-\sqrt{1-z^{-2}}}\right] \quad (8)$$

Fig. 2 shows that the shape factor g_a varies from 0 (infinite flat disk) to 1/3 (sphere) to 1/2 (infinite cylinder). It is noted that g_a values of 0 and 1/3 yield thermal conductivity relationships that are mathematically equivalent to the HSU and HSL bounds given by Hashin and Shtrikman [13]. These bounds can thus be expressed in terms of the weighting factor F as given be Eqs. (9) and (10):

$$F = \frac{1}{3}\frac{k_f}{k_s} + \frac{2}{3} \quad \text{(HSU)} \quad (9)$$

$$F = \frac{3(k_f/k_s)}{1 + 2(k_f/k_s)} \quad \text{(HSL)} \quad (10)$$

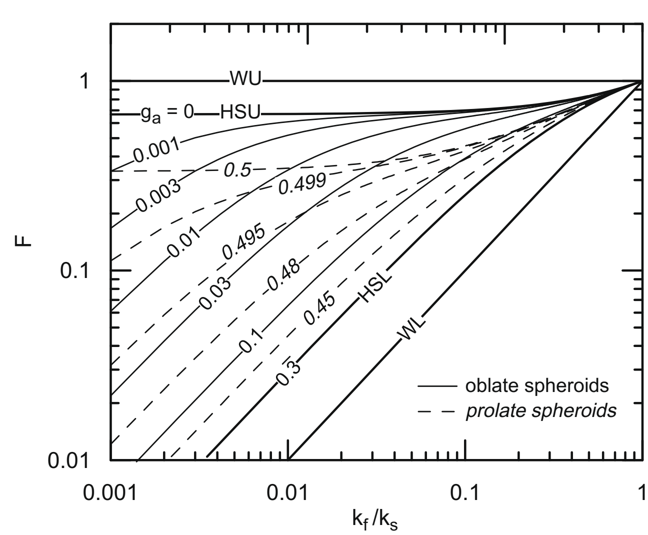

Fig. 3. Expected domain of F values for two-phase porous geomaterials.

It can be mathematically demonstrated that the Wiener bounds can also be expressed in terms of F:

$$F = 1 \quad \text{(WU)} \tag{11}$$

$$F = \frac{k_f}{k_s} \quad \text{(WL)} \tag{12}$$

Fig. 3 shows the F–k_f/k_s relationships for the Weiner and Hashin/Shtrikman bounds and for various values of g_a. It is shown that for a given bound or a given value of g_a, the value of F increases with increasing values of k_f/k_s. The range of values of F is much wider for low values of k_f/k_s than for high k_f/k_s values. This agrees well with the observation made on actual data which showed that the effect of structure increase with decreasing k_f/k_s (or increasing k_s/k_f). It is expected that the values of F for various two-phase geomaterials should lie between the Hashin/Shtrikman bounds or at least between the Weiner bounds.

4.1. Data analyses

The use of Fricke's model [7] to characterize the effect of structure on the thermal conductivity of geomaterials is investigated herein as illustrated with the determination of typical values for F and g_a determined for the thermal conductivity data of Woodside and Messmer [2].

The k–n relationships obtained from Woodside and Messmer [2] are shown in Fig. 4 for (a) porous sandstones and (b) quartz sands particle packs. Both type of materials had the same k_s value equal to 8.5 W m^{-1} K^{-1}. The materials were saturated with air (k_f = 0.024 W m^{-1} K^{-1}), oil (k_f = 0.13 W m^{-1} K^{-1}) and water (k_f = 0.6 W m^{-1} K^{-1}). The porosity of the sandstones ranged from 0.03 to 0.59. The particle size of the sand packs ranged from 74 to 840 μm and the porosity, from 0.2 to 0.59.

The experimental data plotted in Fig. 4 show that for a porosity near 0, the effective thermal conductivity k is about equal to k_s. Increasing porosity leads to a decrease of k. Fig. 4 also shows the effect of structure and of the k_f/k_s ratio on the effective thermal conductivity. For example, a geomaterial made of quartz sand particles with a porosity n of 0.2 and saturated with water (dot), oil (diamond) and air (triangle), the effective thermal conductivity values of 5.16, 2.19 and 0.67 W m^{-1} K^{-1}, respectively. In comparison, the thermal conductivities of sandstone samples (cemented quartz sand particles) are equal to 4.51, 3.76 and 2.40 W m^{-1} K^{-1} showing an increasing effect of structure with decreasing k_f/k_s ratios.

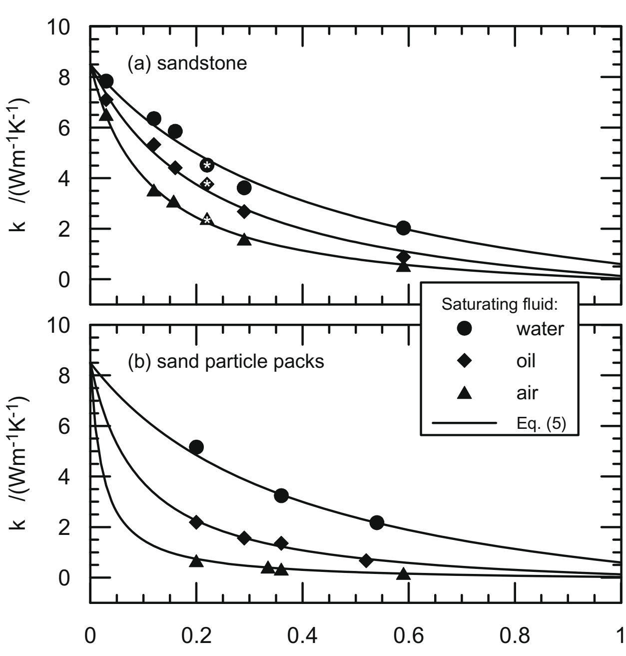

Fig. 4. Best-fit curves to the experimental data of (a) sandstones and (b) sand particle packs saturated with various fluids (data from Woodsinde and Messmer [2]).

Curve fitting of the k–n relationships using Eq. (5) provided the values of F for each material tested. Fig. 4 shows that Eq. (5) provides indeed very good curve-fit (full lines) to the experimental data for both types of materials. The computed F values are equal to 0.290, 0.085 and 0.023 for sand packs and equal to 0.310, 0.190 and 0.100 for sandstone saturated with water, oil and air, respectively. These values of F are plotted as function k_f/k_s in Fig. 5. It is observed that F values systematically increase with increasing k_f/k_s ratios and that distinct relationships were obtained for each type of materials. These results show thus an increasing effect of structure with decreasing k_f/k_s values.

4.2. Applicability of g_a to geomaterials

DeVries [24] extended Fricke's model to three phase porous media such as moist soils. An empirical value of g_a equal to 0.125 appeared to fit fairly well a wide range of data for moist natural soils from [1,3,25]. However, for the same soils, but in dry conditions (two-phase), Johansen [4] demonstrated that a suitable value of g_a would be equal to 0.1, suggesting thus that the values of g_a may not be unique for a given soil. These values of g_a represent disc-like spheroids, which is quite unusual for many soils such as sands and gravels. As shown in Fig. 5, the experimental F–k_f/k_s relationships indeed correspond to several theoretical F–k_f/k_s relationships (Eq. (6)) with g_a values ranging from 0.09 to 0.13. For sandstones, theoretical g_a values range from 0.015 to 0.09.

This clearly demonstrates that the g_a parameter alone cannot account for structure effect of two-phase porous geomaterials. Thus, any extension of the theory behind Fricke's model to these materials should include other parameters to consider the influence of structure on the estimation of k. It was also suggested that the g_a parameter should be regarded as a fitting parameter rather than a strictly theoretically derived geometrical parameter [26]. Instead of developing either alternative, this paper proposes to characterize the structure effects on k using a modified version of the relative thermal conductivity model for moist soils developed by Côté and Konrad [6].

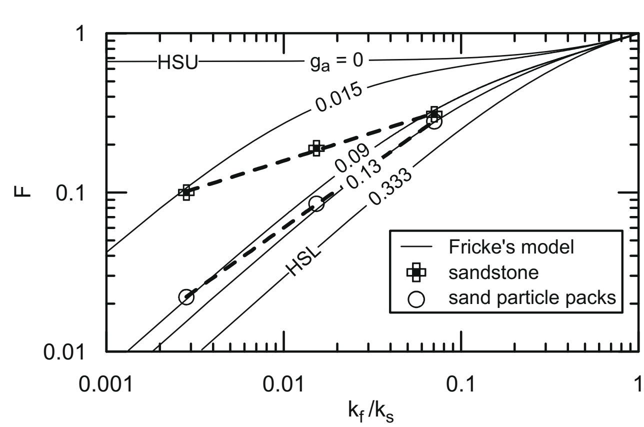

Fig. 5. Variations of F and the corresponding g_a values for the data from Woodside and Messmer [2].

5. The relative thermal conductivity approach

Farouki [27,28] established that the relative thermal conductivity concept developed by Johansen [4] gave the best predicting results for the widest range of moist soils among seven models evaluated. This model normalizes the effective thermal conductivity k of a given soil at given moisture content as a function of the effective thermal conductivity of the soil at the same porosity when saturated with air (minimum value: k_{dry}) and when saturated with water (maximum value; k_{sat}) as expressed by:

$$k_r = \frac{k - k_{dry}}{k_{sat} - k_{dry}} \quad (13)$$

Johansen [4] demonstrated that k_r can be related to the degree of saturation (S_r) of soils and that different k_r–S_r relationships are obtained depending on grain size distribution and the frozen/unfrozen states. Any k_r–S_r relationship should meet the following conditions: (a) $0 \leqslant k_r \leqslant 1$, (b) for $S_r = 0$: $k = k_{dry}$ and $k_r = 0$ and (c) for $S_r = 1$: $k = k_{sat}$ and $k_r = 1$.

5.1. The Côté and Konrad model

Recent research projects in environmental and soil science as well as in geotechnical engineering have recognized the versatility of the relative thermal conductivity concept. Various k_r–S_r relationships have been successfully developed [6,29–31]. Côté and Konrad [6] developed a generalized k_r–S_r relationship (Eq. (14)) where the dimensionless empirical parameter κ is associated with the structure of three-phase porous geomaterials (distribution of grain size, pore size and pore water). The κ parameter is equal to 4.6, 3.55, 1.9 and 0.6 for gravel, sand, silt and clays and peat in the unfrozen state, respectively, and equal to 1.7, 0.95, 0.85 and 0.25 for these geomaterials in the frozen state:

$$k_r = \frac{\kappa S_r}{1 + (\kappa - 1)S_r} \quad (14)$$

By substituting Eq. (14) into Eq. (13) and solving for k, Côté and Konrad [32] proposed Eq. (15) as a generalized thermal conductivity equation:

$$k = \frac{(\kappa k_{sat} - k_{dry})S_r + k_{dry}}{1 + (\kappa - 1)S_r} \quad (15)$$

This concept can also be applied to two-phase porous geomaterials considering that the maximum thermal conductivity is equal to that of the solid phase (k_s) and the minimum thermal conductivity is equal to that of fluid phase (k_f), consequently Eq. (13) can be re-written as:

$$k_r = \frac{k - k_f}{k_s - k_f} \quad (16)$$

In this case, k_r varies with porosity (n) and the conditions are: (a) $0 \leqslant k_r \leqslant 1$, (b) for $n = 0$: $k = k_s$ and $k_r = 1$ and (c) for $n = 1$: $k = k_f$ and $k_r = 0$. Considering an empirical parameter κ_{2P} to account for structure effects in two-phase porous geomaterials, Eq. (14) can be re-written as:

$$k_r = \frac{\kappa_{2P}(1 - n)}{1 + (\kappa_{2P} - 1)(1 - n)} \quad (17)$$

Substituting Eq. (16) into Eq. (17) and solving for k thus leads to an expression of the effective thermal conductivity similar to Eq. (15):

$$k = \frac{(\kappa_{2P} k_s - k_f)(1 - n) + k_f}{1 + (\kappa_{2P} - 1)(1 - n)} \quad (18)$$

While it appears that Eq. (18)is mathematically equivalent to Eq. (5) in Fricke's model, the lines of reasoning behind the parameters F and κ_{2P} of both models are different. The F parameter is a theoretical weighting factor related to the shape of non-touching particles. In the Côté and Konrad model for moist soils, the κ_{2P} parameter is an empirical weighting factor related to the thermal structure of moist soil, which is mostly influenced by the grain contact and pore size distributions, the water and air distribution in the pore space and by the continuity of the three phases. Similarly, in the proposed model for two-phase porous geomaterials (Eq. (18)), it is expected that the κ_{2P} parameter will characterize the continuity of the solid phase and the particle contact density. The values of κ_{2P} will be obtained for several thermal conductivity data sets found in the literature and will be analyzed in terms of the thermal structure of geomaterials made of different types of particles.

5.2. Determination of κ_{2P} for various geomaterials

Thermal conductivity data sets for wide range of two-phase porous geomaterials found in the literature are studied in this paper, using only the data sets for which k_s, k_f and n are known or can be reasonably estimated. The materials are made of (a) natural soil particles, (b) crushed rock particles and (c) porous sedimentary rock, all saturated either with air or with water.

Table 1 summarizes the characteristics of each materials studied including the values of k_s, k_f, the ranges of d, n and k. The experimental k–n relationships are shown in Figs. 6–8 along with the best-fit curves obtained from Eq. (18).

For natural soils saturated with water, thermal conductivity data sets from Ratcliffe [33] yielded a κ_{2P} value equal to 0.45 (Fig. 6a). For air-saturated natural soils from Smith [1], Johansen [4] and Tavman [34], κ_{2P} values of 0.057, 0.040 and 0.037 were obtained, respectively (Fig. 6b).

Thermal conductivity data from Brigaud and Vasseur [35] for packs of crushed dolostone, limestone (1) and limestone (2) saturated with water were studied and the κ_{2P} values 0.34, 0.47 and 0.52 were obtained, respectively (Fig. 7a). The data from Côté and Konrad [36] for packs of crushed quartzite, granite and syenite saturated with air gave κ_{2P} values of 0.075, 0.105 and 0.120, respectively (Fig. 7b).

Porous sedimentary rock from Brigaud and Vasseur [35] (Fig. 8) were also studied and the κ_{2P} values obtained were equal to 0.3, 0.35 and 0.45 for water-saturated sandstone, dolostone and limestone (1), respectively, while κ_{2P} values of 0.095 and 0.125 were obtained for air-saturated sandstone and dolostone, respectively.

6. Discussion

The curves obtained from Eq. (18) in Figs. 6–8 offer fairly good fits to the experimental data analyzed herein this paper, which clearly suggests that this equation describes relatively well the k–n relation taking into account the effect of structure in two-phase porous geomaterials, provided that the thermal conductivity of the solids and the fluids are known.

The values of κ_{2P} obtained for each material analyzed are reported in Table 1 and plotted as a function of k_f/k_s in Fig. 9 (including the data from Woodside and Messmer [2]). The natural soil particles (rounded/sub-rounded) are represented by the empty dots, while the crushed rock particles (angular/sub-angular) and the sedimentary rock (cemented) are represented by the full triangles and the crosses, respectively. Data from Johansen [4] for natural dry soils having a k_s value of 3 W m^{-1} K^{-1} with a corresponding κ_{2P} value of 0.053 is also shown in Fig. 8. For each of the three types of materials studied, the values of κ_{2P} systematically decrease with decreasing k_f/k_s ratios and, as expected, they all plot within the Hashin/Shtrikman bounds.

Table 1
Summary of geomaterials studied

Figure	Reference	Solid/fluid	k_s (W m^{-1} K^{-1})	k_f (W m^{-1} K^{-1})	$k_f/k_s/(10^{-3})$	Measurements, range of			κ_{2P}
						d (μm)	n	k (W m^{-1} K^{-1})	
Natural soils (rounded/sub-rounded particles)									
4b	[2]	Quartz sand/water	8.5	0.600	70.6	74–840	0.2–0.55	2.2–5.2	0.290
4b	[2]	Quartz sand/oil	8.5	0.130	15.3	74–840	0.2–0.52	0.7–2.2	0.085
4b	[2]	Quartz sand/air	8.5	0.024	2.82	74–840	0.2–0.59	0.2–0.7	0.023
6a	[33]	Sediments/water	2.8	0.570	203	0–80	0.4–0.7	0.7–1.4	0.440
6b	[4]	Fine sand/air	3.8	0.024	6.32	80–600	0.25–0.45	0.2–0.4	0.040
6b	[1]	Loam/air	2.9	0.024	8.28	0–80	0.32–0.57	0.1–0.4	0.057
6b	[34]	Sand/air	4.2	0.024	5.71	400–1200	0.36–0.45	0.2–0.3	0.037
Crushed rock (angular/sub-angular particles)									
7a	[35]	Dolostone/water	5.3	0.600	113	0–250	0.26–0.28	2.7–2.9	0.340
7a	[35]	Limestone 1/water	3.3	0.600	181	0–250	0.29–0.32	2.0–2.1	0.470
7a	[35]	Limestone 2/water	2.8	0.600	214	0–250	0.34–0.38	1.6–1.8	0.520
7b	[36]	Quartzite/air	5.0	0.024	4.80	0–20.000	0.15–0.42	0.5–1.4	0.075
7b	[36]	Granite/air	2.6	0.024	9.23	1–20.000	0.18–0.33	0.4–0.9	0.105
7b	[36]	Syenite/air	1.6	0.024	15.0	2–20,000	0.15–0.18	0.6–0.7	0.120
Sedimentary rock (cemented/bound particles)									
4a	[2]	Sandstone/water	8.5	0.600	70.6	–	0.03–0.59	2.0–7.8	0.310
4a	[2]	Sandstone/oil	8.5	0.130	15.3	–	0.03–0.60	0.9–7.1	0.190
4a	[2]	Sandstone/air	8.5	0.024	2.82	–	0.03–0.61	0.5–6.5	0.100
8a	[35]	Sandstone/water	7.7	0.600	77.9	–	0.01–0.44	2.6–7.1	0.300
8a	[35]	Dolostone/water	5.3	0.600	113	–	0.02–0.41	2.1–5.5	0.350
8a	[35]	Limestone 1/water	3.3	0.600	182	–	0.01–0.48	1.6–3.1	0.450
8b	[35]	Sandstone/air	7.7	0.024	3.12	–	0.01–0.30	2.0–6.7	0.095
8b	[35]	Limestone 1/air	3.3	0.024	7.27	–	0.01–0.48	0.4–2.9	0.125

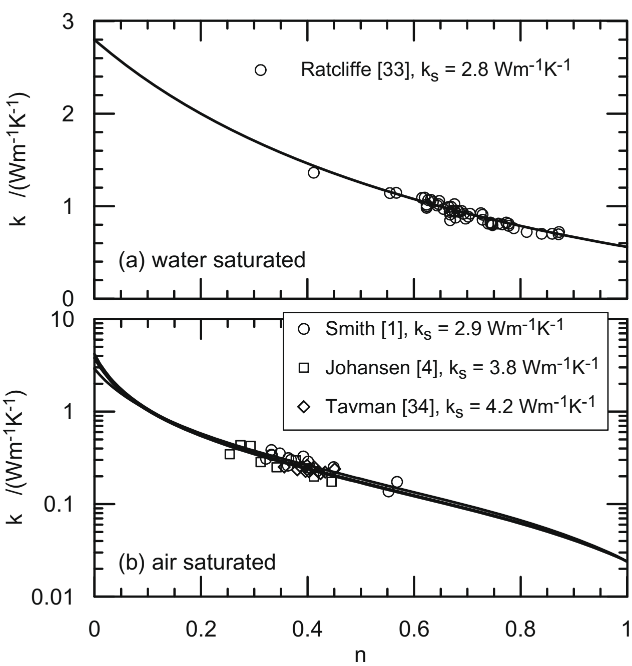

Fig. 6. Best-fit curves to the experimental data of natural soils (rounded/sub-rounded particles) (a) water-saturated and (b) air-saturated.

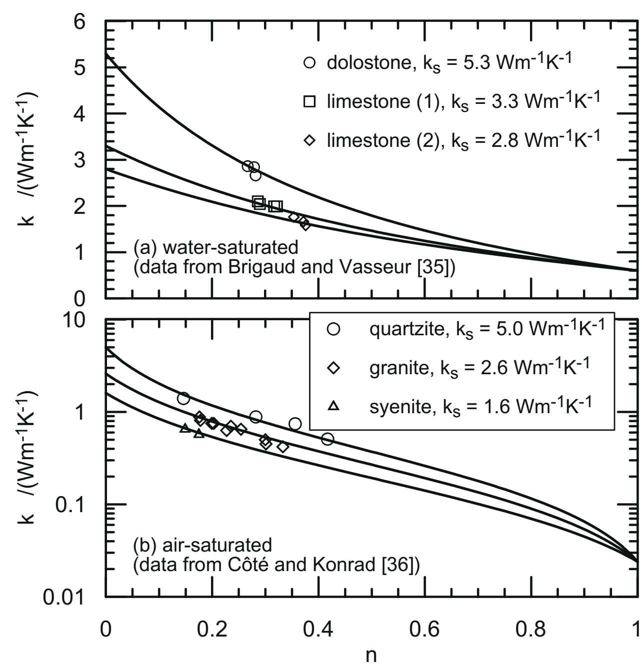

Fig. 7. Best-fit curves to the experimental data of crushed rock (angular/sub-angular particles) (a) air saturated and (b) water saturated.

6.1. Particle shape and cementation

Fig. 9 shows that distinct κ_{2P}–k_f/k_s relationships are obtained for each material type studied. The highest κ_{2P} values were obtained for the cemented particles, intermediate values for the angular/sub-angular particles and the lowest values corresponding to rounded/sub-rounded particles. As expected the κ_{2P}–k_f/k_s relationships obtained show that the natural soils with rounded/sub-rounded particles stand close to the HSL bound (no particle contacts) indicating small contact areas between particles and reduced continuity of the solid phase. The κ_{2P}–k_f/k_s relationship for crushed rock with angular/sub-angular particles stand above that the natural soils indicating better contact areas and increased continuity of the solid phase. The contact areas and the continuity of the solid phase are further increased when bounding agents are presents as evidenced by the κ_{2P} values that are systematically higher than those of the unbound materials. However, this later relationship is still far from the HSU bound characterizing continuous solid phase materials such as foams. In fact, the κ_{2P} values obtained in this study approximately occupy the lower half of the area comprised

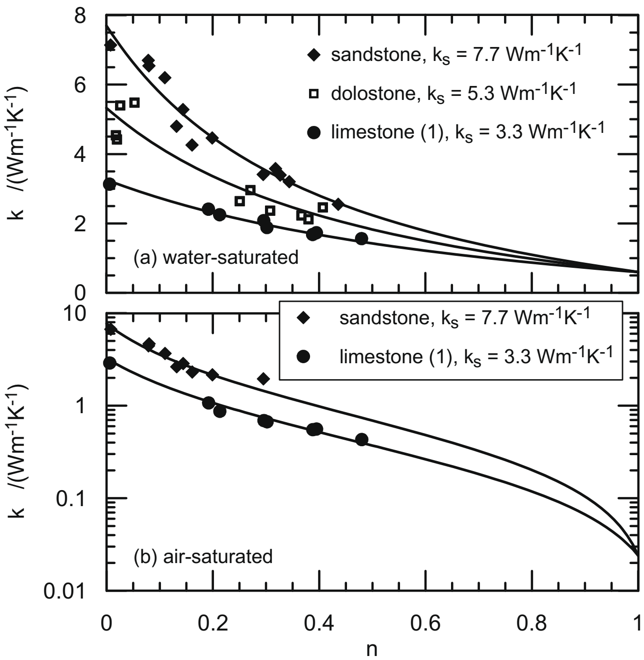

Fig. 8. Best-fit curves to the experimental data sedimentary rock (cemented/bound particles) (a) water-saturated and (b) air-saturated (data from Brigaud and Vasseur [35]).

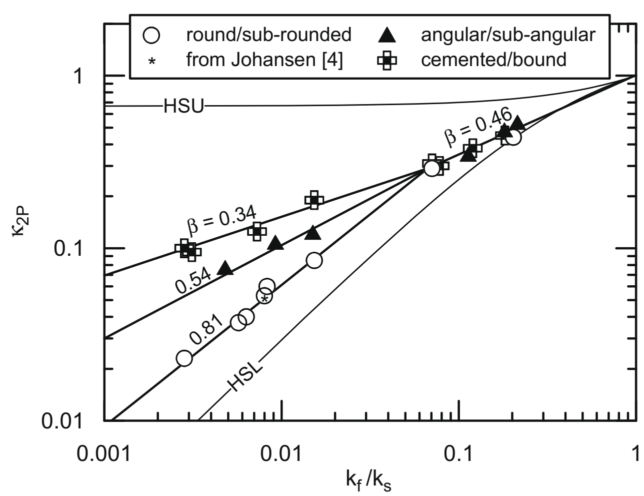

Fig. 9. Values of the structure parameter κ_{2P} for various two-phase porous geomaterials.

between the bounds. Similarly, it was also observed that the effective thermal conductivity of dry soils and rocks generally stand in the lower part of the area limited by the Hashin/Shtrikman bounds [4,15].

The solid trend lines drawn from the κ_{2P} data show that the influence of structure is the highest for the lower values of k_f/k_s and that this influence tends to decrease with increasing k_f/k_s ratios. These trend lines merge at a k_f/k_s ratio of 1/15 (0.067) where the influence of structure on the effective thermal conductivity may be considered as negligible. This is supported from observations in previous studies [2,4,37] where k_f/k_s ratios between 1/15 and 1/10 are often cited as a limiting value at which the structure begins to have an impact on k.

The geomaterials with the rounded/subrounded particles studied in this paper were made of 100% natural soil particles while those with angular/sub-angular particles were made of 100% crushed rock particles giving distinct κ_{2P}–k_f/k_s relationships. Consequently, it is speculated that mixes including different proportions of these two types of particle would give thermal conductivity data leading to intermediate κ_{2P}–k_f/k_s relationships.

6.2. Grain-size and pore-size distributions

The data analyzed herein represent of a wide variety of materials with various particle-size and pore-size distributions. Although the effect of these geotechnical parameters on the effective thermal conductivity were not specifically addressed in this paper, it can be postulated that their effect is not significant since single κ_{2P}–k_f/k_s relationships were obtain for each type of material studied. For example, the natural soils (rounded/sub-rounded) category comprise clays with particle diameter (d) typically ranging from $0 < d < 2\ \mu m$, silts with $2\ \mu m < d < 80\ \mu m$ and sands with $80\ \mu m < d < 5000\ \mu m$ and a single κ_{2P}–k_f/k_s relationship was obtained. Carson et al. [11] also noted that numerical simulations of theoretical soils showed no significant influence of particle size. On the other hand, Tavman [34] observed a slight increase of k when sand particles increase in size. It is, however, possible that the increase in k is caused by higher quartz content, which, in turn, lead to higher k_s values in the geomaterials with larger sand particles [4,5,38,39]. The reader should, however, keep in mind that in three-phase porous geomaterials (solid–liquid–gas) the grain size distribution will influence the water distribution in pore-space according to material type giving thus different thermal bridge effect at the solid-to-solid contacts, which influence the effective thermal conductivity as observed experimentally [3,4,36] and numerically [40].

It is also noted that in gas-saturated materials with very low porosity, the mean pore size may be smaller than the mean free path of saturating gas. The thermal conductivity of such gas should thus not be used to compute the effective thermal conductivity of the porous media [8]. For most "dry" geomaterials, the saturating gas is air at atmospheric pressure, which has a mean free path of $0.066\ \mu m$ [41]. As shown by Woodside and Messmer [2], only 3.5% of the pores of Berea sandstone are smaller than $0.08\ \mu m$. Since most porous geomaterials have fairly broad pore-size distributions with very few pores smaller than $0.1\ \mu m$; this issue has not been addressed in the present paper.

7. A simplified κ_{2P} relationship

The data presented in Fig. 9 suggest that the structure of geomaterials begins to have an impact on k at a limit value of k_f/k_s equal to 0.067 (1/15) for which κ_{2P} is approximately equal to 0.29 for all types of materials. This observation allows defining a simplified expression for the relationships between κ_{2P} and the k_f/k_s ratio plotted by the heavy lines in Fig. 9 and expressed by Eq. (19). The empirical parameter β gives the slope of the κ_{2P}–k_f/k_s relationships in a log–log scale. The values of β for each type of materials are given in Fig. 9 and are also reported in Table 2:

$$\kappa_{2P} = 0.29(15k_f/k_s)^{\beta} \tag{19}$$

Table 2
Values of fitting parameters β of the proposed simplified κ_{2P} model

Materials	k_f/k_s	β
All	>1/15	0.46
Round/sub-rounded	$\leqslant$1/15	0.81
Angular/sub-angular	$\leqslant$1/15	0.54
Cemented/bound	$\leqslant$1/15	0.34

The values of κ_{2P} obtained from Eq. (19) can subsequently be used in Eq. (13) to estimate the effective thermal conductivity of two-phase porous geomaterials as a function the thermal conductivities of each component, porosity and considering the effect of structure which is mostly influenced by the type of particles within the porous media. Natural soils have rounded/sub-rounded particles, crushed rocks have angular/sub-angular particles and sedimentary rocks have cemented particles (rounded or angular). It is therefore expected that the proposed model can be useful to provide fairly simple and accurate estimations of the effective thermal conductivity given that most of the materials used in geotechnical engineering fall within one of the tree categories of materials studied in this paper. This is demonstrated by the validation with independent data given below.

7.1. Validation for glass and lead beads

Fig. 10 shows measured and estimated effective thermal conductivity for (a) homogeneous glass beads and lead beads and (b) Berea sandstone from Woodside and Messmer [2] saturated with gas and liquids of varying thermal conductivity. It is stressed that 15 out of 18 data sets are independent of the present study. The remaining three data sets are identified by a symbol * inserted within the data points in Figs. 10 and 4.

The data are presented in the same manner as in the original paper where the *x*-axis represents the k_s/k_f ratio and the *y*-axis represents the k/k_f ratio. Fig. 10 shows the estimated k functions obtained from the simplified κ_{2P} relationship (Eq. (19), thick black line). A very good agreement between the measured and estimated values is obtained for both the homogeneous glass and lead beads and the Berea sandstone, demonstrating thus that the proposed model can describe the effect of structure of either homogeneous and heterogeneous geomaterials.

7.2. Validation for cement concrete

It is speculated that the proposed κ_{2P} models developed in this study may be applied to estimate the effective thermal conductivity of porous cement concrete which are generally made of crushed rock particles and hydraulic cement such as Portland cement. Analysis of nearly 100 thermal conductivity data sets from the literature [42–46] and from unpublished data of the first author's data bank has thus been performed. The materials studied ranged from lightweight cement concrete and mortar to high density cement concrete. The thermal conductivities of the solid fraction (mineral particles and hydrated cement) were calculated from a method given by Missenard [47]. The κ_{2P} values for each material were obtained from Eq. (19) using the β value of 0.34 proposed for cemented/bound particles. Fig. 11 shows that a very good agreement between the computed thermal conductivities and the measured ones with most of the data standing between the 20% deviation lines. It can therefore be postulated that the structure effect on effective thermal conductivity of industrially cemented geomaterials (crushed and natural particles) is the same than for naturally cemented geomaterials (natural particles) and that the effect of particle shape in cemented/bound materials is negligible.

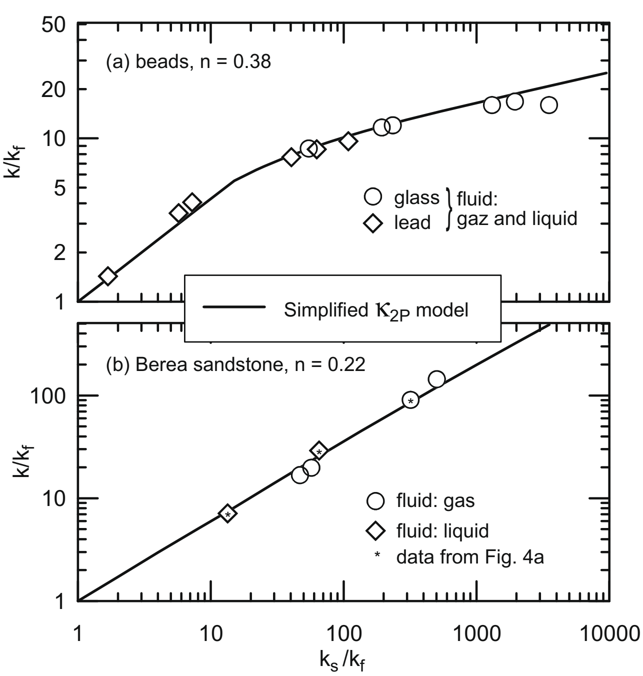

Fig. 10. Comparison between predicted and independent data (except *) of thermal conductivity of (a) packed beads and (b) Berea sandstone (data from Woodside and Messmer [2]).

8. Conclusion

This paper presented a study on the effect of structure on the effective thermal conductivity for 21 solid–fluid combinations representative of two-phase porous geomaterials. A total of 193 thermal conductivity data sets from the literature for natural soil, crushed rocks and sedimentary rocks were analyzed.

It was shown that the theoretical model from Fricke's model [7] gave good fit to data sets from Woodside and Messmer [2]. However, any theoretical extension of the model should include other parameters to account for structure effect. The relative thermal conductivity approach developed by Johansen [4] and further improved by Côté and Konrad [6] was thus retained and modified for two-phase porous geomaterials. It was demonstrated that the new structure parameter κ_{2P} is mathematically equivalent to the shape factor F in Fricke's model. It was found that the κ_{2P} parameter describing the shape of the k–n relationship decreases with decreasing k_f/k_s ratios. The results of this study clearly demonstrated that the effect of structure is noticeable at k_f/k_s ratios lower than 1/15 and increases as the k_f/k_s ratio decreases. Distinct κ_{2P}–k_f/k_s relationships were obtained for each material type studied. The study showed no significant effect of particle-size and pore-size distributions effects on the effective thermal conductivity of two-phase porous geomaterials.

A simplified κ_{2P}–k_f/k_s relationship was proposed to assess the effect of structure on the effective thermal conductivity of geomaterials. The κ_{2P} parameter is estimated as a function of the k_f/k_s ratios and the empirical parameter β related to the structure of

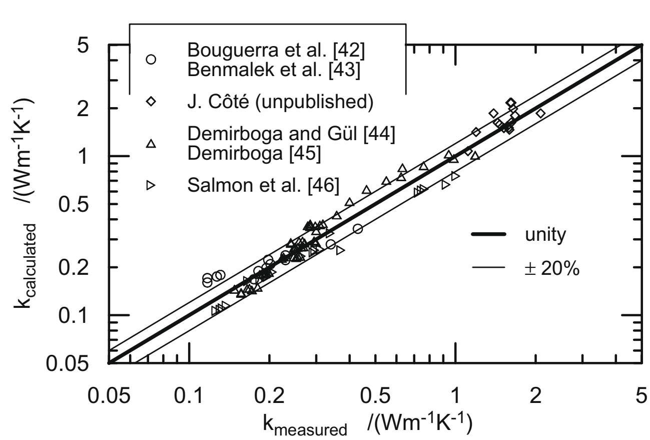

Fig. 11. Comparison between predicted and independent data of thermal conductivity of lightweight and conventional cement concrete.

geomaterials. The new relationship applied well to nearly 120 independent thermal conductivity data sets from homogeneous bead packs, sedimentary rock and cement concrete. The data for sedimentary rock and cement concrete showed that both type of materials behave in the same manner.

Acknowledgments

This research was sponsored by the Natural Science and Engineering Research Council of Canada (NSERC), through an individual grant and the industrial research chair on the operation of infrastructures submitted to frost action (CREIG).

References

[1] W.O. Smith, The thermal conductivity of dry soil, Soil Sci. 53 (1942) 425–459.
[2] W. Woodside, J.M. Messmer, Thermal conductivity of porous media, J. Appl. Phys. 32 (9) (1961) 1688–1706.
[3] M.S. Kersten, Laboratory Research for the Determination of the Thermal Properties of Soils, Research Laboratory Investigations, Engineering Experiment Station, University of Minnesota, 1949.
[4] O. Johansen, Varmeledningsevne av jordarter, Ph.D. Thesis, Norge tekniske hogskole, Trondheim, Norway, 1975.
[5] J. Sundberg, Thermal Properties of Soils and Rock, Swedish Geotechnical Institute, Report No. 35, Linköping, Sweden, 1988.
[6] J. Côté, J.M. Konrad, A generalised thermal conductivity model for soils and construction materials, Can. Geotech. J. 42 (2005) 443–458.
[7] H. Fricke, A mathematical treatment of the electric conductivity and capacity of disperse systems I: the electric conductivity of a suspension of homogeneous spheroids, Phys. Rev. 24 (1924) 575–587.
[8] M. Kaviany, Principles of Heat Transfer in Porous Media, second ed., Springer, New York, 1995. pp. 119–132.
[9] J.K. Mitchell, Fundamentals of Soil Behavior, second ed., Wiley, New York, 1993. pp. 131–142.
[10] R.L. Hamilton, O.K. Crosser, Thermal conductivity of heterogeneous two-component systems, Ind. Eng. Chem. Fund. 1 (3) (1962) 187–191.
[11] J.K. Carson, S.J. Lovatt, D.J. Tanner, A.C. Cleland, An analysis of the influence of material structure on the effective thermal conductivity of theoretical porous materials using finite element simulations, Int. J. Refrig. 26 (2003) 873–880.
[12] I. Taguchi, M. Kurashige, Microstructures and macroscopic conductivity of randomly packed and uniaxially pressed sphere compacts, Int. J. Heat Mass Transfer 50 (2007) 3809–3819.
[13] Z. Hashin, S. Shtrikman, A variational approach to the theory of the effective magnetic permeability of multiphase materials, J. Appl. Phys. 33 (1962) 3125–3131.
[14] J.C. Maxwell, A Treatise on Electricity and Magnetism, Clarendon Press, Oxford, 1873.
[15] J.K. Carson, S.J. Lovatt, D.J. Tanner, A.C. Cleland, Thermal conductivity bounds for isotropic, porous materials, Int. J. Heat Mass Transfer 48 (2005) 2150–2158.
[16] N. Phan-Tien, G.W. Milton, New bounds on the effective thermal conductivity of *N*-phase materials, Proc. Roy. Soc. Lon. Ser. A 380 (1982) 333–348.
[17] M.N. Miller, Bounds for effective electrical, thermal, and magnetic properties of heterogeneous materials, J. Math. Phys. 10 (1969) 1988–2004.
[18] A.S. Mickley, Thermal conductivity of moist soil, Am. Inst. Elec. Eng. Trans. 70 (1951) 1789–1797.
[19] R.A. Crane, R.I. Vachon, A prediction of the bounds on the effective thermal conductivity of granular materials, Int. J. Heat Mass Transfer 20 (1977) 711–723.
[20] G.R. Hadley, Thermal conductivity of packed metal powders, Int. J. Heat Mass Transfer 29 (1986) 909–920.
[21] P.K. Samantray, P. Karthikeyan, K.S. Reddy, Estimating effective thermal conductivity of two-phase materials, Int. J. Heat Mass Transfer 49 (2006) 4209–4219.
[22] J. Wang, J.K. Carson, M.F. North, D.J. Cleland, A new approach to modelling the effective thermal conductivity of heterogeneous materials, Int. J. Heat Mass Transfer 49 (2006) 3075–3083.
[23] R.C. Progelhof, J. L Throne, R.R. Ruetsch, Methods for predicting the thermal conductivity of composite systems: a review, Polym. Eng. Sci. 16 (9) (1976) 615–625.
[24] D.A. De Vries, Thermal properties of soils, in: W.R. Van Wijk (Ed.), Physics of Plant Environment, North-Holland, Amsterdam, 1963, pp. 210–235.
[25] W.O. Smith, H.G. Byers, The thermal conductivity of dry soils of certain of the great soil group, Am. Soc. Soil Sci. Proc. 3 (1938) 13–19.
[26] G.S. Campbell, J.D. Jungbauer Jr., W.R. Bidlake, R.D. Hungerford, Predicting the effect of temperature on soil thermal conductivity, Soil Sci. 158 (1994) 307–313.
[27] O.T. Farouki, Thermal Properties of Soils, CRREL Monograph 81-1, United States Army Corps of Engineers, Cold Regions Research and Engineering Laboratory, Hanover, New-Hampshire, 1981.
[28] O.T. Farouki, Evaluation of Methods for Calculating Soil Thermal Conductivity, CRREL Report 82-8, United States Army Corps of Engineers, Cold Regions Research and Engineering Laboratory, Hanover, New-Hampshire, 1982.
[29] V. Tarnawski, W.H. Leong, K.L. Bristow, Developing a temperature-dependant Kersten function for soil thermal conductivity, Int. J. Energy Res. 24 (2000) 1335–1350.
[30] V. Balland, P.A. Arp, Modeling soil thermal conductivities over a wide range of conditions, J. Environ. Eng. Sci. 4 (2005) 549–558.
[31] S. Lu, T. Ren, R. Horton, An improved model for predicting soil thermal conductivity from water content at room temperature, Soil Sci. Soc. Am. J. 71 (2007) 8–14.
[32] J. Côté, J.M. Konrad, Estimating the thermal conductivity functions of granular materials and subgrade soils, Transport. Res. Rec. 1967 (2006) 10–19.
[33] E.H. Ratcliffe, The thermal conductivity of ocean sediments, J. Geophys. Res. 65 (5) (1960) 1535–1541.
[34] I.H. Tavman, Effective thermal conductivity of granular porous materials, Int. Commun. Heat Mass Transfer 23 (2) (1996) 169–176.
[35] F. Brigaud, G. Vasseur, Mineralogy, porosity and fluid control on thermal conductivity of sedimentary rocks, Geophys. J. 98 (1989) 525–542.
[36] J. Côté, J.M. Konrad, Thermal conductivity of base-course materials, Can. Geotech. J. 42 (2005) 61–78.
[37] J.H. Sass, A.H. Lachenbruch, R.J. Munroe, Thermal conductivity of rocks from measurements on fragments and its application to heat-flow determinations, J. Geophys. Res. 76 (14) (1971) 3391–3401.
[38] O. Johansen, Beregningsmetode for varmeledningsevnen av fuktige of frosne jordarter, Det II, Frost. I. Jord. 10 (1973) 13–28.
[39] J. Côté, J.M. Konrad, Indirect methods to assess the thermal conductivity of Quebec marine clay solid particles, Can. Geotechn. J. 44 (2007) 1117–1127.
[40] P. Lehman, M. Stähli, A. Papritz, H. Fhühler, A fractal approach to model soil structure and to calculate thermal conductivity of soils, Transport Porous Med. 52 (2003) 313–332.
[41] D.R. Lide, Handbook of Chemistry and Physics, 76th ed., CRC Press, New York, 2003. pp. 14.14–14.21.
[42] A. Bouguerra, A. Ledhem, F. de Barquin, R.M. Dheilly, M. Quéneudec, Effect of microstructure on the mechanical and thermal properties of lightweight concrete prepared from clay, cement and wood aggregates, Cement Concrete Res. 28 (1998) 1179–1190.
[43] M.L. Benmalek, A. Bouguerra, A. Ledhem, R.M. Dheilly, M. Quéneudec, Caractéristiques de bétons légers à base de résidus d'exploitation de carrières et de bois, Can. J. Civil Eng. 26 (1999) 374–377.
[44] R. Demirboga, Influence of mineral admixtures on thermal conductivity and compressive strength of mortar, Energy Buildings 35 (2003) 189–192.
[45] R. Demirboga, R. Gül, The effects of expanded perlite aggregate silica fume and fly ash on the thermal conductivity of lightweight concrete, Cement Concrete Res. 33 (2003) 723–737.
[46] D.R. Salmon, R.G. Williams, R.P. Tye, Thermal conductivity and moisture measurements on masonry materials, in: A.O. Desjarlais, R.R. Zarr (Eds.), Insulation Materials: Testing and Applications, vol. 4, ASTM STP 1426, American Society for Testing and Materials, West Conshohocken, PA, 2002.
[47] A. Missenard, Conductivité thermique des solides, liquides, gaz et de leurs mélanges, Éditions Eyrolles, Paris, 1965, pp. 307–320.